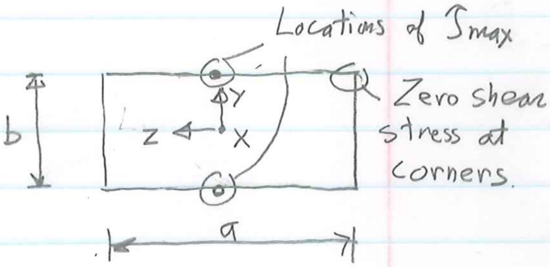
14. Twisting of solid rectangular shaft

In general, solutions for rectangular shafts have to be obtained by solving the differential equations of elasticity (as well as for more arbitrarily shaped cross-sections).

Consider the rectangular cross-section, shown at right. Results are usually given in the following form:



$$\tau_{max} = \frac{T}{C_1 a b^3} \quad , \quad \frac{T}{\phi} = \frac{JG}{L}$$

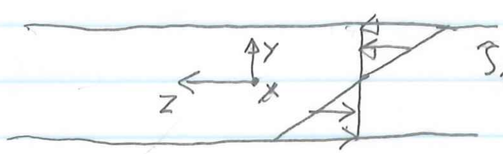
where $J = C_2 a b^3$

The coefficients C_1 and C_2 depend on the ratio a/b:

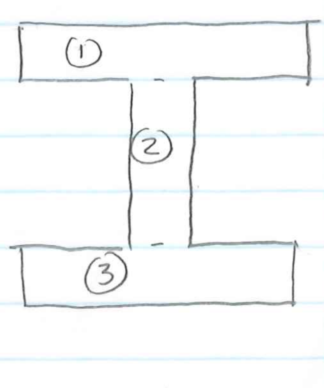
a/b	C_1	C_2
1.0	0.208	0.141
2.0	0.246	0.229
3.0	0.267	0.263
5.0	0.291	0.291
10.0	0.312	0.312

$\rightarrow$ both approach $\frac{1}{3}$ as $\frac{a}{b} \to \infty$.

If the cross-section is thin ($a \gg b$), then the shear stress is primarily τ_{xz} except near the end, and varies linearly through the thickness.



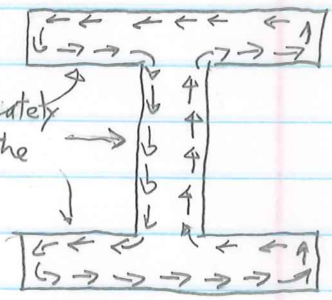
Often, cross-sections consists of several rectangular plates, such as shown at right for an I section. The J for this shaft would be the sum of the individual J_i values.



$$J = J_1 + J_2 + J_3$$

The total torque is the sum of that carried by each plate: $T = T_1 + T_2 + T_3$
where $T_i = T \cdot J_i / J$ for $i = 1, 2, 3$.
To find τ_{max} for plate i, use T_i and that plate's a and b values.
A picture of the shear stress distribution appears at right.

Varies approximately linearly through the thickness.



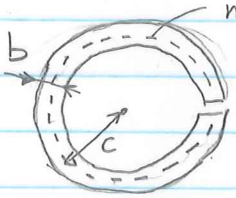
The formulas for a single plate can also be applied to a sliced tube where the wall thickness is small compared to the diameter.

Thus,

$$J = \frac{1}{3} a b^3 = \frac{1}{3} 2\pi c t^3$$

thickness = b = t

mid-thickness length = a = $2\pi c$

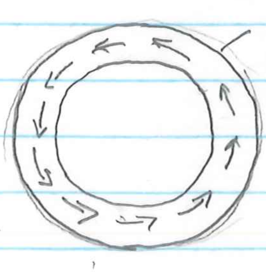


This can be compared to a closed tube with the same diameter and thickness using the formula on page 29. For the closed tube

$$J = 4 A^2 / \oint ds/t = 4(\pi c^2)^2 / \frac{2\pi c}{t} = 2\pi c^3 t$$

The ratio $\dfrac{J_{\text{closed tube}}}{J_{\text{sliced tube}}} = 3 \dfrac{c^2}{t^2}$ shows that the twisting stiffness of the closed tube will be much greater than that for the sliced tube.

Note also the difference in shear stress distribution.



Shear stress is constant through the thickness.

Shear stress varies linearly through the thickness.

Warping is especially pronounced for a sliced tube.